

$\Phi \in \mathcal{H}_1 \otimes \mathcal{H}_2$, $\dim \mathcal{H}_i = 2$.

Assertion: A n.d.s. cond. for

$\forall P_2 \exists P_1 \ni \langle P_1 P_2 \rangle_\Phi = \langle P_1 \rangle_\Phi \neq 0$
is that Φ is entangled.

Suppose Φ entangled: $\Phi = \alpha_1 u_1 \otimes v_1 + \alpha_2 u_2 \otimes v_2$

$\{u_i\}$ o.n., $\{v_i\}$ o.n., $|\alpha_1|^2 + |\alpha_2|^2 = 1$, $\alpha_i \neq 0$.

Given P_2 there is a basis $w_1, w_2 \ni$

$$P_2 w_1 = w_1, \quad P_2 w_2 = 0.$$

$$v_1 = c_1 w_1 + c_2 w_2$$

$$v_2 = d_1 w_1 + d_2 w_2$$

c_1, d_1 cannot both be 0,
for then orthogonality of
 v_1, v_2 fails.

$$\begin{aligned} \Phi &= (\alpha_1 c_1 u_1 + \alpha_2 d_1 u_2) \otimes w_1 \\ &\quad + (\alpha_1 c_2 u_1 + \alpha_2 d_2 u_2) \otimes w_2. \end{aligned}$$

$$\alpha_1 c_2 u_1 + \alpha_2 d_2 u_2 \neq 0.$$

Let P_1 be the projector on \mathcal{H}_1 which has
this vector as eigenvector with eigen-
value 1. The vectors orthogonal to it have
eigenvalue 0; let Z_1 be such.

$$\begin{aligned} \langle P_1 P_2 \rangle_\Phi &= |(Z_1, \alpha_1 c_1 u_1 + \alpha_2 d_1 u_2)|^2 \\ &= \langle P_1 \rangle_\Phi \neq 0. \end{aligned}$$

If Φ is not entangled, α_1 or α_2 is 0.
Say α_1 . Then there can be a P_2 such
that $\alpha_1 c_1 u_1 + \alpha_2 d_1 u_2 = 0$. Then for
any P_1 , $\langle P_1 P_2 \rangle_\Phi = 0$.